Tomographic Imaging of Cross-Well EM Data

EM4.4

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SUMMARY

A 2-D iterative EM Born inversion method has been developed for estimating the Earth's electrical conductivity structure between two boreholes. The theory is developed for a vertical magnetic dipole located in a bore hole in either a whole-space or half-space that exhibits a cylindrical symmetry about the bore hole axis. The inversion scheme uses the first order Born approximation to invert for an initial model, and then employs higher order terms in the Born series to achieve more accurate images.

The inversion routine has been applied to a cross-hole data set collected at the British Petroleum test site near Devine, Texas. The geology is predominantly 1-D with interbedded layers of sandstone, shale and limestone. The resulting image shows a resistive zone within a conductive medium which is consistent with both the well log and 1-D inversions.

Continuing experiments are being conducted at the University of California's Richmond Field Station test site. The goal of these experiments is to use cross-well EM to monitor the injection of salt water plumes. Inversion results of forward model calculations that simulate the experiment show that the iterative Born method does reconstruct the horizontal extent of the plume. Initial tests of the inversion using the field data support these model results.

INTRODUCTION

Due to recent improvements in field instrumentation and computing power, considerable attention has been given to cross-hole EM induction as a method to analyze the conductivity structure of the earth. The research has been devided into two categories: 1) developing a field system which can accurately measure the down-hole EM fields generated by a magnetic dipole source in a second borehole some distance away and 2) developing inversion routines to obtain high resolution images of subsurface electrical properties. In 1989 researchers from U.C. Berkeley, Lawrence Berkeley Laboratory (LBL) and Lawrence Livermore National Laboratory (LLNL) began to jointly address these problems. To this date several sets of cross-well data have been collected at various locations across the U.S. using the system developed by the group. However, only recently have inversion routines been successfully applied to data collected in these experiments.

ITERATIVE BORN INVERSION FORMULATION

The imaging scheme employed here uses the first order Born approximation as a first step. The theory follows that proposed by Zhou (1989) and Sena and Toksoz (1990) except the problem is formulated in the space domain rather than in the wave-number domain. For a cylindrically symmetric medium which has conductivity variations in the r and z directions (Figure 1), the integral equation for the scattered EM fields due to a vertical magnetic dipole source can be written as

$$Hz^S(r_r,z_r;r_t,z_t) = \int_{z'}\int_{r'} (\sigma_a(r',z')-\sigma_o)E(r',z';r_t,z_t)G_{Hz}(r_r,z_r;\ r',z')dr'dz' \qquad (1)$$

where Hz^S is the measured secondary vertical magnetic field at receiver position (r_r,z_r) due to a transmitter at position (r_t,z_t), σ_a and σ_o are the anomalous and background conductivities, E is the total electric field in the ϕ direction which can be expressed as the sum of the primary and secondary fields, i.e., $E = E^P + E^S$, and G_{Hz} is the vertical magnetic field Green's function for the background medium. Equation (1) is highly non-linear between Hz^S and $(\sigma_a(r',z')-\sigma_o)$ due to the fact that the electric field, E, is dependent on the anomalous conductivity. Thus the estimation of this parameter is a nonlinear process requiring extensive computer time and memory which currently allows only the simplest problems to be analyzed. However when the medium contains only weak scatterers, i.e. when $(\sigma_a(r',z')-\sigma_o)$ is small, then a linearized version of (1) ensues through the use of the first order Born approximation, $E \approx E^P$(Kong,1975):

$$Hz^S(r_r,z_r;r_t,z_t) = \int_{z'}\int_{r'} (\sigma_a(r',z')-\sigma_o)E^P(r',z';r_t,z_t)G_{Hz}(r_r,z_r;\ r',z')dr'dz'. \qquad (2)$$

This equation is now linear between Hz^S and $(\sigma_a(r',z')-\sigma_o)$which implies that the scattering currents within the inhomogeneities are negligibly small and thus coupling between individual scatterers can be neglected. By discretizing the medium into N cells across which both the anomalous conductivity and primary electric field are assumed constant (Hohmann,1971), equation (2) for the first-order Born approximation becomes

$$Hz^S(r_r,z_r;r_t,z_t) = \sum_{j=1}^{N} (\sigma_a(r_j,z_j)-\sigma_o)E^P(r_j,z_j;r_t,z_t)\int_{z'}\int_{r'} G_{Hz}(r_r,z_r;\ r',z')dr'dz'. \qquad (3)$$

If the electrical properties are known (for example from borehole logs) then E^P and G_{Hz} can be estimated and the anomalous conductivity can be inverted for using a least squares inversion technique.

The iterative process is implemented by first approximating the secondary electric fields at the center of each pixel due to the inhomogenous conductivity (σ_a) that was solved for in (3) above. This estimation is accomplished by using the iterative Born series approximation for electric fields, i.e.,

$$E^S_i(r_k,z_k;r_t,z_t) = \sum_{j=1}^{N} (\sigma_a(r_j,z_j)-\sigma_o)\left(E^P(r_j,z_j;r_t,z_t)+E^S_{i-1}(r_j,z_j;r_t,z_t)\right)\int_{z'}\int_{r'} G_{Hz}(r_k,z_k;\ r',z')dr'dz' \qquad (4)$$

where E^S_i is the calculated i'th order Born approximation of the electric field and E^S_{i-1} is the value calculated in the previous iteration

(Note: For i=1, $E^S_{i-1} = 0$). The i'th order Born inversion is completed by using these calculated secondary electric fields in the equation

$$Hz^S(r_r, z_r; r_t, z_t) = \sum_{j=1}^{N} (\sigma_a(r_j, z_j) - \sigma_o) \tag{5}$$

$$\left(E^P(r_j, z_j; r_t, z_t) + E^S_{i-1}(r_j, z_j; r_t, z_t)\right) \int_{z'} \int_{r'} G_{Hz}(r_r, z_r;\ r', z')\,dr'dz'.$$

Notice that equation (5) is nonlinear in $(\sigma_a(r_j, z_j) - \sigma_o)$. The conductivity is once again estimated using least squares inversion, and equations (4) and (5) are iterated upon until convergence occurs.

Initial results using point conductors and slab like bodies as the forward models show improvement over Zhou's(1989) method. The iterative technique results in increased spatial resolution and it is able to handle larger conductivity contrasts better than the noniterative Born approximation. This indicates that coupling between individual scattering currents can not be neglected.

INVERSION OF THE DEVINE DATA SET

In September of 1990, the first set of cross-well EM measurements were collected with the LBL/LLNL system at the British Petroleum test site near Devine, Texas (Wilt et al, 1991). Examination of the well logs indicates that the geology at the site consists of interbedded sandstones, shales and limestones which are continuous and flat lying across the area. Resistivities of the rocks range from 1 to 300 Ohm-m.

The survey was conducted in two boreholes separated by 100m and lined with plastic from 160m to the bottom at 900m. The cross borehole survey was conducted from a depth of approximately 549m to 669m spanning a series of 2 to 3 ohm-m sands and shales to a 30m thick, 10ohm-m section of limestone and back to sands and shales. The data were collected by fixing the receiver location in one borehole and then moving the transmitter over the same 120m section in the other well while continuously making measurements. 13 profiles at 512Hz and 2048Hz were obtained in this manner with approximately 8m between successive receiver positions. Figure 2 shows an expanded section of the well log from one of the holes with a 1-D interpretation of the EM data.

The image resulting from the inversion of the 500 Hz data is shown in Figure 3. Notice that although the image does not show perfectly flat lying beds, it does show a resistive area separating two conductive zones, the positions of which coincide very well with the well log and 1-D inversion. Because the integral equation theory is developed for isolated bodies rather than extensive conductivity anomalies, we feel that these results are excellent. Inversions of the 2048Hz data and simultaneous inversions of both frequencies show similar results.

THE RICHMOND FIELD STATION INJECTION EXPERIMENT

We are conducting salt water injection experiments at the University of California's Richmond Field Station 7 miles north of the UCB campus. The geology consists of approximately 35 to 40 meters of interbedded clays, silts, sands and gravels overlying more resistive metamorphic basement. Four 70m PVC cased monitoring wells are arranged roughly in a square of diagonal 50m with a 70m deep injection well located at the center of the square. This well is perforated at 30m to allow for the injection of 40,000 gallons of conductive salt water into a 3m aquifer. Cross-well data are being collected both before and after injection with the transmitter in the center hole and the receiver at various positions in each of the outer monitoring wells. This yields a total of eight cross-well data sets from which an image of the plume will be constructed using the iterative Born inversion technique.

In preparation for the field test we calculated some simple models of an injected salt water plug. Figure 4a shows a 3m thick, cylindrically symmetric, conductive plume extending out 5m from the borehole, while Figure 5a has the plume extending out 8m. The results for these models were calculated for a frequency of 18.8KHz using an integral equation solution developed by Zhou (1989). The inversion results (Figures 4b and 5b) definitely show that the extent of plume can be detected using cross-borehole EM induction. Initial imaging attempts using the first set of Richmond Field Station data show similar characteristics.

CONCLUSIONS

An iterative Born inversion technique has been developed which incorporates higher order terms of the Born series to achieve better images of the Earth's conductivity than a simple first order Born approximation. The imaging scheme has been proven to work on cross-well field data with some limitations due to the size of the inhomogeneities present. It has also been shown that the method is very useful in monitoring the injection of fluids into an aquifer and the extent to which a plume has migrated.

ACKNOWLEDGMENT

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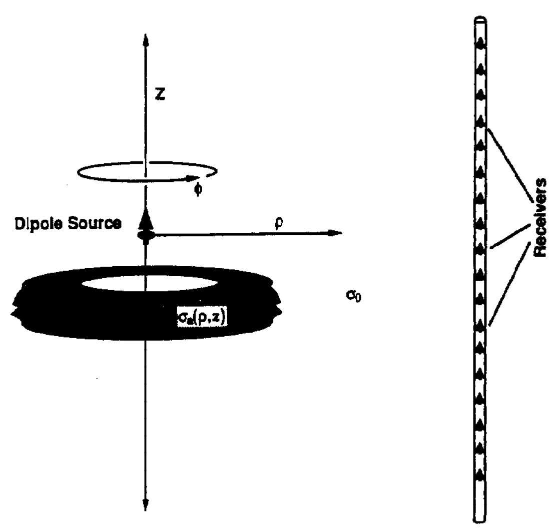


Figure 1. 2-D Earth with cylindrical symmetry.

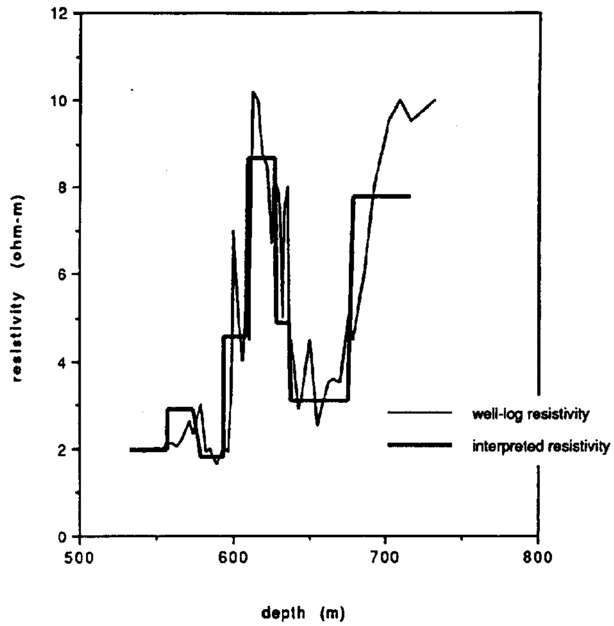


Figure 2. Comparison of a borehole induction log from the Devine test site and the interpreted 1-D cross-hole EM results.

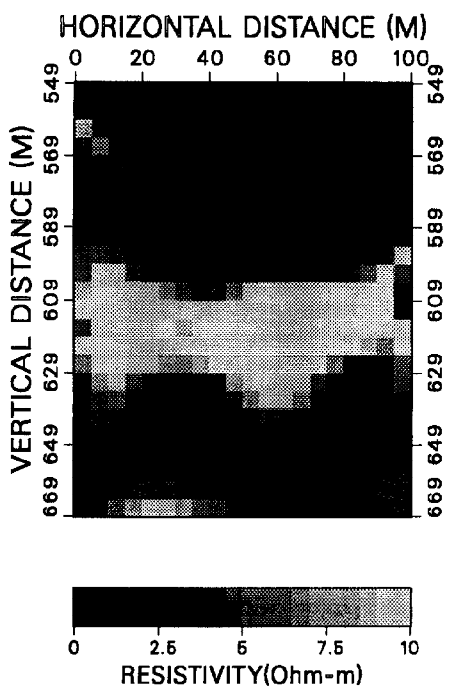


Figure 3. Iterative Born inversion of Devine test site data.

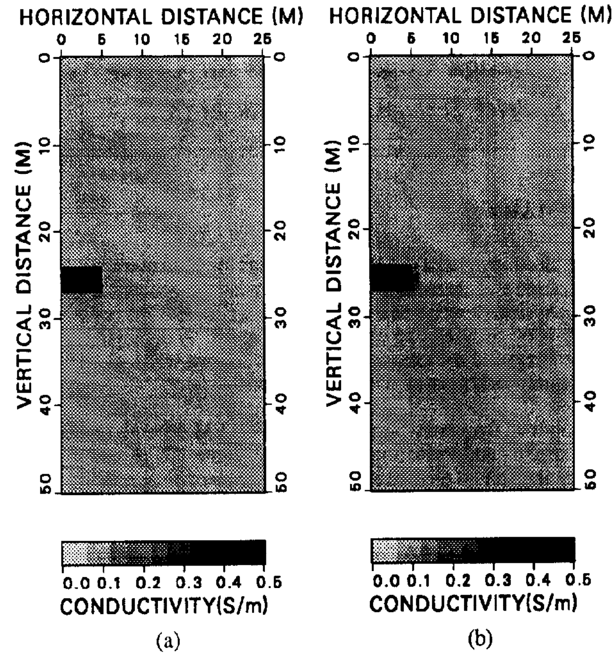


Figure 4. a) First model used to simulate the Richmond experiment. The plume is cylindrically symmetric about the borehole and extends out 5m. b) Iterative Born inversion of the model results.

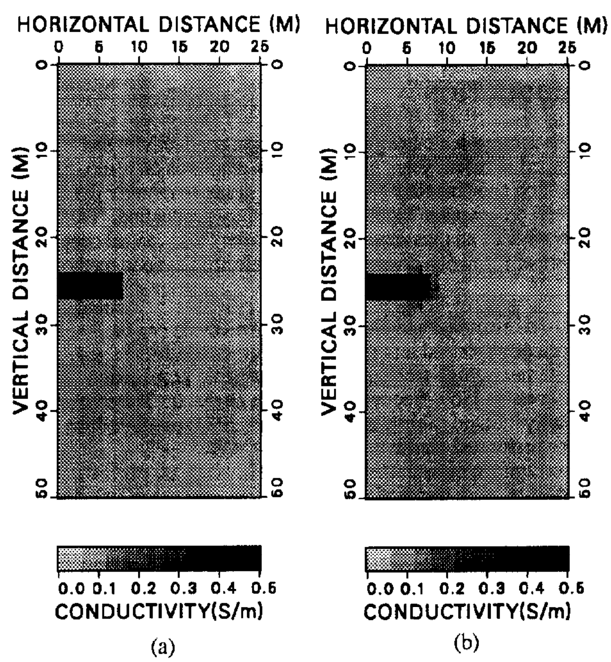
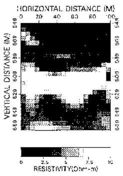
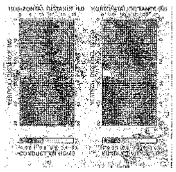


Figure 5. a) Second model used to simulate the Richmond experiment. The plume is cylindrically symmetric about the borehole and extends out 8m. b) Iterative Born inversion of the model results.


HORIZONTAL DISTANCE (M)
VERTICAL DISTANCE (M)
RESISTIVITY(Ohm-m)


HORIZONTAL DISTANCE (M)
VERTICAL DISTANCE (M)
CONDUCTIVITY(S/M)


HORIZONTAL DISTANCE (ft)
HORIZONTAL DISTANCE (ft)